Lecture 2: Coulomb's Law

Coulomb's law

If two charged particles are brought near each other, they each exert a force on the other. If the particles have the same sign of charge, they repel each other (Figs. 21-6*a* and *b*). That is, the force on each particle is directed away from the other particle, and if the particles can move, they move away from each other. If, instead, the particles have opposite signs of charge, they attract each other (Fig. 21-6*c*) and, if free to move, they move closer to each other. This force of repulsion or attraction due to the charge properties of objects is called an **electrostatic force.** ***The force between two charges at a distance, r apart is directly proportional to the product of the two charges and inversely proportional to the square of the distance between them***

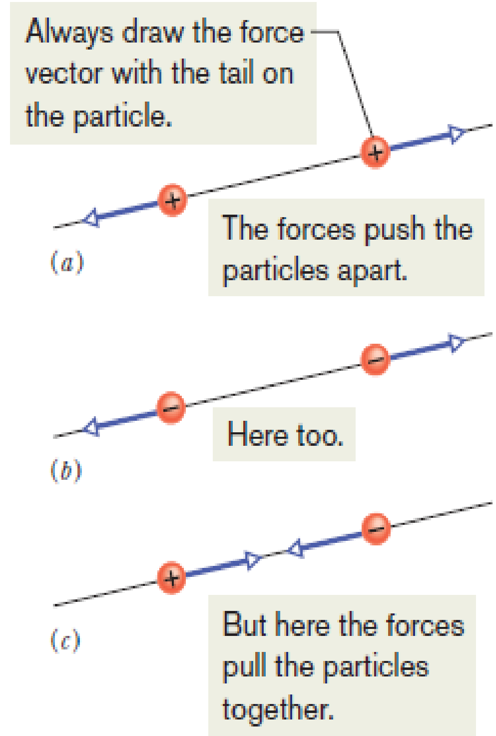


Fig. 21-6 Two charged particles repel each other if they have the same sign of charge, either (*a*) both positive or (*b*) both negative. (*c*) They attract each other if they have opposite signs of charge.

The equation giving the force for charged *particles* is called **Coulomb's law** after Charles-Augustin de Coulomb, whose experiments in 1785 led him to it. In terms of the particles in Fig. 21-7, where particle 1 has charge q_1 and particle 2 has charge q_2, the force on particle 1 is mathematically wriiten as

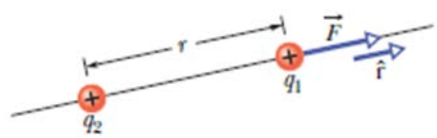


Fig. 21-7 The electrostatic force on particle 1 can be described in terms of a unit vector r̂ along an axis through the two particles.

$$F = \frac{q_1 q_2}{4\pi\varepsilon_\circ r^2} = K\frac{q_1 q_2}{r^2} \qquad (1)$$

where, $K = \dfrac{1}{4\pi\varepsilon_\circ} \approx 9.0\times10^9\,\mathrm{Nm^2C^{-2}}$ = a constant and $\varepsilon_\circ$ is permitivity of free space.

$$\varepsilon_0 = 8{,}85\cdot10^{-12}\,F/m \qquad \mathrm{C^2/N.m^2}$$

Curiously, the form of Eq. 21-1 is the same as that of Newton's equation (Eq. 13-3) for the gravitational force between two particles with masses m_1 and m_2 that are separated by a distance r:

$$\vec{F} = G\frac{m_1 m_2}{r^2}\hat{\mathbf{r}} \qquad \text{(Newton's law)}, \qquad (21\text{-}2)$$

in which G is the gravitational constant.

But electric and gravitational interactions are two distinct classes of phenomena. Electric interactions depend on electric charges and can be either attractive or repulsive, while gravitational interactions depend on mass and are always attractive (because there is no such thing as negative mass).

(a) Figure 21-8*a* shows two positively charged particles fixed in place on an x axis. The charges are $q_1 = 1.60 \times 10^{-19}$ C and $q_2 = 3.20 \times 10^{-19}$ C, and the particle separation is $R = 0.0200$ m. What are the magnitude and direction of the electrostatic force $\vec{F}_{12}$ on particle 1 from particle 2?

Two particles: Using Eq. 21-4 with separation R substituted for r, we can write the magnitude F_{12} of this force as

$$F_{12} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{R^2}$$

$$= (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2) \times \frac{(1.60 \times 10^{-19}\ \text{C})(3.20 \times 10^{-19}\ \text{C})}{(0.0200\ \text{m})^2} = 1.15 \times 10^{-24}\ \text{N}.$$

Thus, force $\vec{F}_{12}$ has the following magnitude and direction (relative to the positive direction of the x axis):

$$1.15 \times 10^{-24}\ \text{N} \quad \text{and} \quad 180°. \qquad \text{(Answer)}$$

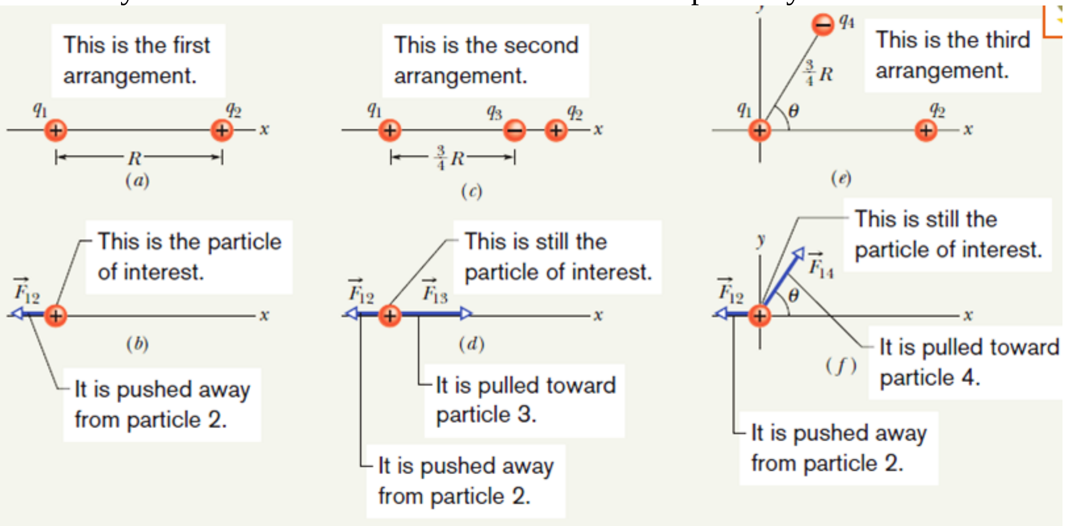


Fig. 21-8 (*a*) Two charged particles of charges q_1 and q_2 are fixed in place on an x axis. (*b*) The free-body diagram for particle 1, showing the electrostatic force on it from particle 2. (*c*) Particle 3 included. (*d*) Free-body diagram for particle 1. (*e*) Particle 4 included. (*f*) Free-body diagram for particle 1.

(b) Figure 21-8*c* is identical to Fig. 21-8*a* except that particle 3 now lies on the x axis between particles 1 and 2. Particle 3 has charge $q_3 = -3.20 \times 10^{-19}$ C and is at a distance $\frac{3}{4}R$ from particle 1. What is the net electrostatic force $\vec{F}_{1,\text{net}}$ on particle 1 due to particles 2 and 3?

Three particles: To find the magnitude of $\vec{F}_{13}$, we can rewrite Eq. 21-4 as

$$F_{13} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_3|}{(\frac{3}{4}R)^2}$$

$$= (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2) \times \frac{(1.60 \times 10^{-19}\ \text{C})(3.20 \times 10^{-19}\ \text{C})}{(\frac{3}{4})^2(0.0200\ \text{m})^2}$$

$$= 2.05 \times 10^{-24}\ \text{N}.$$

The net force $\vec{F}_{1,net}$ on particle 1 is the vector sum of $\vec{F}_{12}$ and $\vec{F}_{13}$; that is, from Eq. 21-7, we can write the net force $\vec{F}_{1,net}$ on particle 1 in unit-vector notation as

$$\vec{F}_{1,net} = \vec{F}_{12} + \vec{F}_{13}$$
$$= -(1.15 \times 10^{-24}\,\text{N})\hat{i} + (2.05 \times 10^{-24}\,\text{N})\hat{i}$$
$$= (9.00 \times 10^{-25}\,\text{N})\hat{i}. \qquad \text{(Answer)}$$

Thus, $\vec{F}_{1,net}$ has the following magnitude and direction (relative to the positive direction of the x axis):

$$9.00 \times 10^{-25}\,\text{N} \quad \text{and} \quad 0°. \qquad \text{(Answer)}$$

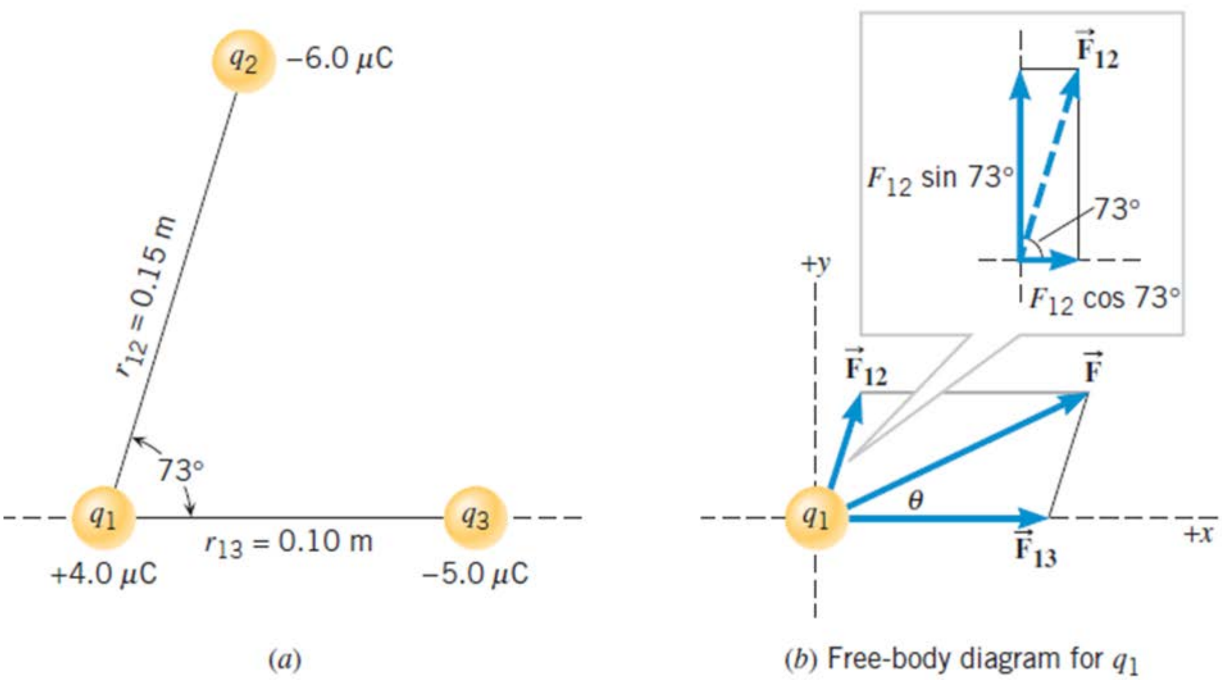


Figure 18.13a shows three point charges that lie in the x, y plane in a vacuum. Find the magnitude and direction of the net electrostatic force on q_1.

Reasoning The force exerted on q_1 by q_2 is $\vec{\mathbf{F}}_{12}$ and is an attractive force because the two charges have opposite signs. It points along the line between the charges. The force exerted on q_1 by q_3 is $\vec{\mathbf{F}}_{13}$ and is also an attractive force. It points along the line between q_1 and q_3. Coulomb's law specifies the magnitudes of these forces. Since the forces point in different directions (see Figure 18.13*b*), we will use vector components to find the net force.

Solution The magnitudes of the forces are

$$F_{12} = k\frac{|q_1||q_2|}{r_{12}^2} = \frac{(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(4.0 \times 10^{-6}\ \text{C})(6.0 \times 10^{-6}\ \text{C})}{(0.15\ \text{m})^2} = 9.6\ \text{N}$$

$$F_{13} = k\frac{|q_1||q_3|}{r_{13}^2} = \frac{(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(4.0 \times 10^{-6}\ \text{C})(5.0 \times 10^{-6}\ \text{C})}{(0.10\ \text{m})^2} = 18\ \text{N}$$

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

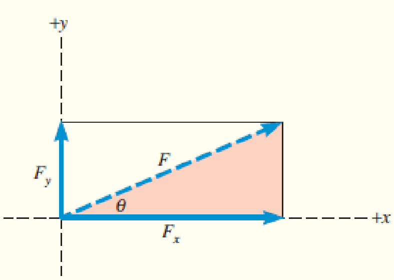


Figure 18.14 Math Skills drawing.

Force	x component	y component
$\vec{\mathbf{F}}_{12}$	$+(9.6\ \text{N})\cos 73° = +2.8\ \text{N}$	$+(9.6\ \text{N})\sin 73° = +9.2\ \text{N}$
$\vec{\mathbf{F}}_{13}$	$+18\ \text{N}$	$0\ \text{N}$
$\vec{\mathbf{F}}$	$\vec{\mathbf{F}}_x = +21\ \text{N}$	$\vec{\mathbf{F}}_y = +9.2\ \text{N}$

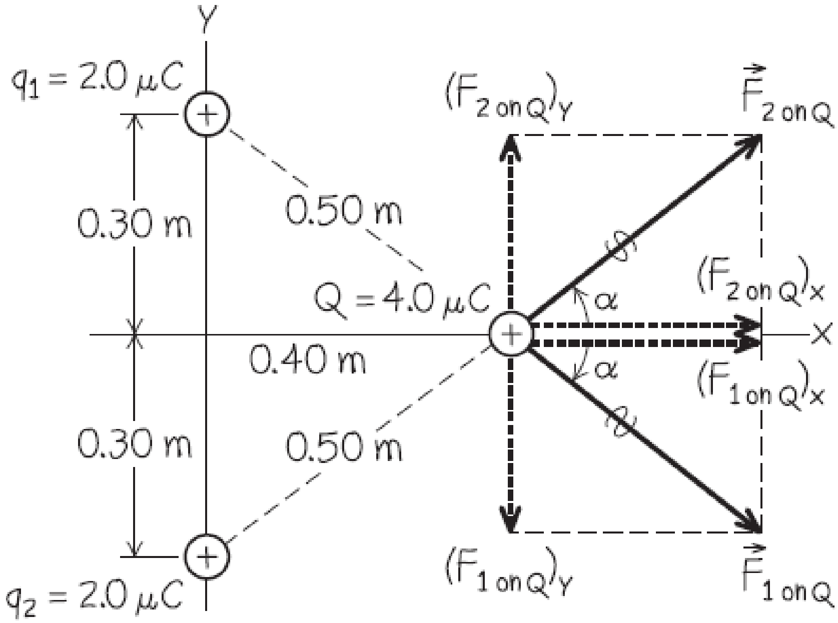
The magnitude F and the angle θ of the net force are

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(21\ \text{N})^2 + (9.2\ \text{N})^2} = \boxed{23\ \text{N}}$$

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{9.2\ \text{N}}{21\ \text{N}}\right) = \boxed{24°}$$

Quiz 1

Two equal positive charges $q_1 = q_2 = 2.0\ \mu\text{C}$ are located at $x = 0$, $y = 0.30$ m and $x = 0$, $y = -0.30$ m, respectively. What are the magnitude and direction of the total electric force that q_1 and q_2 exert on a third charge $Q = 4.0\ \mu\text{C}$ at $x = 0.40$ m, $y = 0$?



EXECUTE: Figure 21.14 shows the forces $\vec{F}_{1\text{ on }Q}$ and $\vec{F}_{2\text{ on }Q}$ due to the identical charges q_1 and q_2, which are at equal distances from Q. From Coulomb's law, *both* forces have magnitude

$$F_{1\text{ or }2\text{ on }Q} = (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2) \times \frac{(4.0 \times 10^{-6}\ \text{C})(2.0 \times 10^{-6}\ \text{C})}{(0.50\ \text{m})^2} = 0.29\ \text{N}$$

The x-components of the two forces are equal:

$$(F_{1\text{ or }2\text{ on }Q})_x = (F_{1\text{ or }2\text{ on }Q})\cos\alpha = (0.29\ \text{N})\frac{0.40\ \text{m}}{0.50\ \text{m}} = 0.23\ \text{N}$$

From symmetry we see that the y-components of the two forces are equal and opposite. Hence their sum is zero and the total force $\vec{F}$ on Q has only an x-component $F_x = 0.23\ \text{N} + 0.23\ \text{N} = 0.46\ \text{N}$. The total force on Q is in the $+x$-direction, with magnitude 0.46 N.

EVALUATE: The total force on Q points neither directly away from q_1 nor directly away from q_2. Rather, this direction is a compromise that points away from the *system* of charges q_1 and q_2. Can you see that the total force would *not* be in the $+x$-direction if q_1 and q_2 were not equal or if the geometrical arrangement of the changes were not so symmetric?